IN MEMORY OF PROFESSOR DANIE KRIGE
WINFRED ASSIBEY-BONSU
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Part 1:  
The Great man - Professor Danie Krige

Part 2:  
A summary of the basic tenets of evaluating the Mineral Resource assets of mining companies, as observed in Professor Krige’s pioneering work over half a century

Conclusion
Part 1 – The Great Man

Professor Daniel Gerhardus Krige
26 August 1919 – 2 March 2013
A World Figure

- Professor Danie Krige passed away on 2\textsuperscript{nd} March 2013 at the age of 93.

- His death was recorded in Wikipedia under notable deaths – a distinction shared with famous persons such as Margaret Thatcher – he was a true giant.
This presentation would be incomplete without throwing light on some of the things Professor Krige held very dear in his life – taken from his 2012 interview with Professor R. C. A. Minnitt of the University of the Witwatersrand;

Professor Krige was a devout Christian, who always emphasised that what made a difference in his life was his belief in Jesus Christ;

He also acknowledged that he had been the recipient of gifts of grace from the Creator - “grace given to him”;

He drew attention to six specific areas in which he could identify the grace of the Almighty at work in his life and career.
The Six ‘Gifts of Grace’

1. A tribute to his parents for the practical application of a godly lifestyle, the establishment of a firm foundation, and a life philosophy that was modelled by them in every area of life. An example being, that even with the limited resources of a pastor, his parents saw to it that seven of the nine siblings received a tertiary education;

2. The second of the gifts of grace that he acknowledged, was the support he had received from his two spouses. He was happily married for 45 years to his first wife (until her death), and thereafter for 20 years to his second wife Ansie;

3. The third gift of grace was the way in which his career developed, and the various changes in direction that it took, as his research unfolded;

4. The fourth gift of grace was that when he returned to work at Anglovaal, they began to apply his advanced methods of evaluation on their mines;

5. The fifth gift of grace was that on retirement from Anglovaal at the age of 60, he received the unexpected opportunity of taking up the Chair of Professor of Mineral Economics at the University of the Witwatersrand, which he occupied for the next 10 years. This enabled him to teach and undertake extensive consulting work for mining companies both locally and internationally, and was, in his opinion, a great blessing;

6. The final gift of grace was that after leaving the University of the Witwatersrand, he was still able to undertake extensive national and international consulting work, which kept him occupied and young for the following 20 years.

He also acknowledged, with deep gratitude, that while the opportunities presented themselves to him, it was his responsibility to make good use of them, and that without these gifts of grace, his life’s work would not have been possible.
Professor Krige and Family (1930)
The Family Man
Professor Krige’s 90th Birthday with Ansie
The Early Years

• Professor Krige matriculated from Monument High School, South Africa at the age of 15;

• In 1938 at the age of 19, he graduated as a Mining Engineer from the University of the Witwatersrand (“Wits University”);

• It was clear early on, that he was destined for great achievements.

• The photographs that follow show the difference between the robe of a university graduate and typical clothes of an underground miner. It provides a perfect illustration of Professor Krige’s values regarding theoretical developments aimed at solving practical problems.
The Early Years
Practitioner, Researcher and Teacher

• Professor Krige worked with Anglo Transvaal on a number of gold mines in the Witwatersrand until 1943;

• He then joined the Government Mining Engineering Department, where he worked for a further 8 years, and spent time studying data and developing mathematical models;

• He returned to industry as Group Financial Engineer of the Anglovaal Group until 1981, when he “retired”;

• He then spent another 10 years as Professor of Mineral Economics at the University of the Witwatersrand.
Initial World Recognition

• Professor Krige’s seminal papers published in the Journal of Chemical, Metallurgical and Mining Society of South Africa, led to additional fundamental research in France on “regionalized variables” by Professor George Matheron and his team;

• Professor Matheron named the new method of linear estimation of the regionalized variables using a spatial model, “Kriging”, in recognition of Professor Krige’s distinguished pioneering work;

• Kriging is currently applied worldwide in the fields of exploration, ore evaluation, environmental studies, petroleum, agriculture, fisheries and other disciplines.
Topics, Content and Effect of Papers

• The statistical explanation of conditional biases in block evaluation;

• It stimulated the use of regression corrections for routine ore reserve evaluations by several mines;

• This technique, in effect, was the first elementary basis of what is now known as “Kriging”;

• The paper introduced, inter alia, the basic geostatistical concepts of “support”, “spatial structure”, “selective mining units” and “grade-tonnage curves”;

• The concept of recoverable resources/reserves in current use is based on what is known as “Krige’s relationship”.
Significant Further Contributions

Major inputs in the fields of investment, financial analysis and taxation, evidenced from various contributions, including:

- the establishment of the original South African uranium contracts;

- by a substantial number of local and overseas publications in this field;

- the publication in 1955, in Afrikaans, of what was probably one of the first papers on risk analysis for new mining investment.
Academic Achievements and Awards

- DSc (Eng) 1963, University of the Witwatersrand
- DIng (HC) 1981, Honorary Degree, University of Pretoria
- Honorary Doctorate Degree from Moscow State Mining University
- Honorary Doctorate Degree from University of South Africa (UNISA)
- Order of Meritorious Service Class 1, Gold, by South African State President
- Southern African Institute of Mining and Metallurgy (SAIMM) – The highest award – Brigadier Stokes Award, 1984
- Many other merit awards from SAIMM including two gold medals in 1966 and 1980; and two silver medals in 1979 and 1993
- International Association of Mathematical geology – William Krumbein medal, 1984
- One of the highest awards from the American Society of Mining Engineers – The Daniel Jackling Award
- Several awards from APCOM International Council, including Distinguished achievement Award, 1989
- Elected as Foreign Associate of US National Academy of Engineers (NAE) 2010, the first South African to ever receive this award for his distinguished contributions to Engineering
- Order of the Baobab in silver – awarded by President Jacob Zuma
Academic Achievements and Awards
Commitment to APCOM

• Professor Krige was South Africa’s representative on the International APCOM Council from its inception;

• He served as the Chairman of the International APCOM council, the 1st non-USA member to be elected to this position;

• He initiated and was directly involved with all arrangements for the Symposia held in South Africa in 1972, 1987 and 2003;

• As far as is known, Professor Krige attended all APCOM Symposia until he was almost 90 years old. In 2003, two weeks after a major operation, he managed to convince his medical doctors to allow him to attend the 2003 APCOM in Cape Town, South Africa, where he was a keynote speaker and also presented two other papers.
APCOM and The Great Wall of China
Continued as a Professor of Mineral Economics at the University of the Witwatersrand, where he was responsible for postgraduate courses in geostatistics and mineral economics, and supervised many masters and doctoral theses;

Presented courses in geostatistics and/or lectured at local South African and also international universities in Australia, Germany, Taiwan, Chile, Russia and China, to name but a few;

Conducted consultancy work locally and internationally;

Participated in and contributed to many international congresses in South Africa, USA, Canada, Germany, Spain, Chile, Colombia, Slovenia, Australia, UK, Russia, France and China.
Publications

• Published some 96 technical papers;

• A 1951 paper, based on his MSc (Eng.) thesis at the University of the Witwatersrand, expounded his pioneering work in geostatistics in more detail;

• Published Geostatistics Monograph, 1st in the series of the SAIMM;

• A complete record of all his publications is available digitally from the SAIMM.
Dedicated Service

• As a Professional Engineer, Professor Krige served for many years on the mining committee of the Engineering Council of South Africa;

• Served on the Council of SAIMM for many years;

• A co-founder of the International Association of Mathematical Geology, GASA, GAA and the Statistical Association of South Africa;

• Served as a director of several companies;

• Served on the sub-committee of the South African Prime Minister’s Economic Advisory Council during 1967/8;

• Designed the state aid formula, which assisted a large number of gold mines to survive the period of low gold prices;

• Served on various committees of the South African Chamber of Mines;

• Member of the SAMREC Working Committee for Reporting Code as first published in 2000.
Part 2 – The Basic Tenet's

Content:

- Motivation – Capital intensiveness of mining
- Data Integrity
- Geology models
- Change of Support
- Efficiency of Estimates
- Conditional bias and Smoothing Effect
- Proposed Tools for the practitioner
Fundamental Assets of mining companies

- Mineral Resources and Reserves are the fundamental assets of mining companies;
- The intensive capital investments basically revolve around the fundamental Mineral Resource and Mineral Reserve assets in the ground – the strategic objective being to explore/acquire/develop and ultimately mine them;
- One of the critical risks exists in the uncertainty of the estimation thereof;
- After intensive capital investment, if it is subsequently found that the expected Mineral Resource and Mineral Reserves were inefficiently estimated or valued, billions of dollars may be lost;
- It is within these critical risks to capital investment, that Professor Krige’s pioneering and research over half a century provides technical solutions to mitigate these.
Capital Intensiveness of Mining

Acquisition, mine development or Expansion cost (US$ billions)

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<tr>
<th>Company</th>
<th>Year</th>
<th>Cost (US$ billions)</th>
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<tr>
<td>Rio Tinto</td>
<td>2007</td>
<td>38.1</td>
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<td>Billiton</td>
<td>2014</td>
<td>27</td>
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Capital Intensiveness of mining

Acquisition, mine development or Expansion cost (US$ billions)

Gold Fields Ltd (F2007) 2.5
Gold Fields Ltd (2011) 1.1
Newmont (2011) 2.3
Barrick (2012) 8.5
Data Integrity

• Quality data is the lifeblood of evaluation;

• Disastrous errors and critical risks include using bad data (‘GIGO - Garbage-In-Garbage-Out’).

Important aspects to consider include:

• Data types and collation processes, including survey processes;
• Ensure data quality;
• Data validation and authorisation (QA/QC);
• Use of approved laboratories;
• Use of standards and blanks;
• Database safety and security;
• Protocols and systems should be in place;
• Sarbanes-Oxley Act of 2002 (SOX) compliance.
Geology models

- Geology is the foundation of mineral resource and reserve modelling;
- Geology should always be recognized as a vital element in deposit modelling;
- Experience has shown that geostatistical assessment without proper geological input can be disastrous and will constitute a critical risk;
- Different ore bodies will behave differently, however the main geology essentials include the lithological and structural features, a model of the origin and formation of the ore body, sedimentology, waste units, geo-zones / geo-domains, density, mineralogy, as well as controls of mineralisation;
- Further considerations include, but are not limited to, alteration, deportment, geochemical anomalies, karsting and water constraint issues, deleterious elements, acid drainage issues, geo-metallurgical test-work, and geotechnical data;
- Various geology environments, with different mineralisation controls, require various types of geology models.
Geology Modelling (Porphyry Cu & Au Ore body)

- A view of the deposit showing geological domains – Cerro Corona, South America
‘Geo’ in Geostatistics

A robust geological model is therefore a pre-requisite, and if the geostatistical model does not agree with the geological one, there are grounds for serious concern. Either one or both models should be critically re-examined so as to establish the essential correlation and validation.

A very dangerous practice is that of subdividing the ore body, not on geological grounds, but directly on grade only. This can lead to serious biases, particularly where the data in one or more subdivisions are insufficient to allow proper geostatistical analyses.

Technique selection and optimal application is paramount.
Historical Background

• In the field of mineral resource and reserve evaluation, geology and geostatistics are the two inseparable sides of the same coin;

• As highlighted previously, on the one side geology concentrates on the physical features of the ore body, such as structures, source, deposition and type of mineralisation;

• Geostatistics is the other side of the coin and provides mathematical, statistical and geostatistical models for the analytical sampling data available, in order to introduce efficient evaluation techniques for resource and reserve estimates, and to attach confidence limits to these;

• Uncertainty is fundamental to all branches of science and to human life itself. It is of particular significance in mineral resource and reserve estimation;

• It is the reason for the introduction of mathematical statistical techniques in geology and to the birth of geostatistics over half a century ago.
Frequency Distribution

• The initial efforts in applying classical statistical procedures to ore evaluation in South Africa date back to 1919 (Watermeyer) and 1929 (Truscott);

• It was only in the late 1940’s and early 1950’s that Sichel (1947, 1952) introduced the lognormal model for gold values, and using this model developed the ‘t’ estimator;

• Departures from the usual lognormal model were largely overcome with the introduction in 1960 (Krige, 1960) of the 3-parameter lognormal, which requires an additive constant before taking logarithms;

• However, there were still cases which could not be covered by the 3-parameter lognormal, and Sichel (1992) introduced the more flexible compound lognormal distributions, originally developed by him for diamond distributions.
Professor’s Danie Krige and Herbert Sichel
The Birth of Geostatistics

• Geostatistics as such did not really originate until the basic concept of ore grades as a spatial variable with a spatial structure, was introduced in 1951/2 by Professor Krige;

• This arose firstly in Professor Krige’s endeavour to explain the experience seen on the South African gold mines for many decades, of ore reserve block estimates during subsequent mining consistently showing significant undervaluation in the lower grade categories and the reverse for estimates in the higher grade categories, i.e. what is now known as conditional biases;

• His pioneering work provided the geostatistical explanation of conditional biases as unavoidable errors resulting from the use of limited data on the periphery of blocks, which were used in valuing ore reserve blocks.
Explanation of Conditional Biases
Spatial Concept and the Birth of “Kriging”

- Professor Krige proposed and implemented corrective measures to eliminate these significant conditional biases;

- The regression corrections were applied routinely to block estimates on several mines in the early 1950’s and represented the actual birth of “Kriging”;

- The regressed estimate was in effect, a weighted average of the peripheral estimate and the global mean of the mine section, it was the first application of Kriging. It can be labeled ‘simple elementary Kriging’, being based on the spatial correlation between the peripheral values and the actual grades of the ore inside the blocks, and giving proper weight to the data outside the block periphery via the mean;

- In this way, the spatial concept and “Kriging” was introduced by Professor Krige;

- The concept of Support is very basic to geostatistics and was first covered by Ross (1950) and further developed by Krige (1951), including Krige’s variance-size of area relationship.
Spatial Structure and Variograms

• By 1963/66 (Krige, 1963, 1966) the spatial patterns were defined in far more detail;

• These studies covered the spatial correlations between individual ‘point’ sample values, as well as that between regularized data blocks;

• The corresponding correlograms or covariograms were used on a Simple Kriging basis for block evaluations;

• Kriging on a routine basis for ore reserve evaluation was therefore, already in use on some Anglovaal gold mines more than 50 years ago.
Spatial Structure and Variograms

• Professor Krige’s pioneering work in early 1950’s aroused interest world wide, particularly in France where, under Professor Allais, Professor Krige’s papers were republished in French (Krige, 1955);

• One of Professor Allais students, later to become world renowned as Professor Matheron, started the development of the theory of Regionalised Variables;

• Matheron, also then proposed the use of the variogram to define the spatial structure. This model is an extension and refinement of the concept covered by De Wijs (1951/3);

• Professor Krige’s regressed estimates were then still called ‘weighted moving averages’ until Matheron’s insistence in the mid-1960’s on the term Kriging in recognition of Professor Krige’s pioneering work.
Conditional Unbiasedness

- It is instructive to observe that on the South African gold mines, the improvement in the standard of block evaluations due to the *elimination of conditional biases*, accounts for some 70% of the total level of improvement achievable today, using the most sophisticated geostatistical techniques. *It is for this reason that Professor Krige placed so much stress on the implementation of conditional unbiasedness;*

- For this reason, it is critical that the elimination of conditional biases is not only the major contributor to the reduction of uncertainty in assessing the mineral resource assets of mining companies, but also an integral and fundamental part of any kriging and mineral resource and reserve assessment process;

- The balance of this part of the presentation will, therefore, concentrate on this vital aspect in a little more detail.
What Contributes to Conditional Biases

• It is a common knowledge concept that any increase in knowledge and available data relevant to any uncertainty being studied, will reduce the level of uncertainty, provided the knowledge is applied properly;

• Knowledge will never be perfect and data never complete, and therefore uncertainty will never be entirely eliminated;

• However, any procedure or technique which does not use all relevant data in order to provide the “best” perspective on the remaining uncertainty, must not be accepted;

• Unfortunately, Professor Krige reported that in his worldwide experience, he encountered many cases where practitioners have erred in respect of this fundamental concept - In too many cases Mineral Resources and Mineral Reserves were estimated on limited data, and relevant data was ignored.
What Contributes to Conditional Biases

1. **Insufficient data.** As discussed in this presentation, in 1950 it was the use of only peripheral data for each block. Today, with geostatistics, the data search routine is still often inadequate, even with the complete data base available to the computer;

2. **No advance analysis** to determine the minimum search routine required to eliminate the biases;

3. **No follow-up studies** to record the presence of these biases and the need to eliminate them.
Conditional Biases – Testing Tools

Mineral Resources for a new or an existing Mine covers two major stages:

• **At the initial or first stage** the data is limited and is obtained either from a broad drill hole grid or from an initial main development grid;

• **During the second or final stage** more data becomes available from grade control or from stope faces and auxiliary developments.

Apart from providing a basis for short and longer term mine planning and viability studies, evaluations are frequently required to provide resource and reserve classification figures (Measured / Indicated / Inferred, and Proven / Probable respectively), to substantiate major capital investments, and / or the raising of loans.
Conditional Biases – Testing Tools

• At both stages of evaluation, the evaluation technique should ensure minimum error variances / uncertainty;

• These requirements are linked closely to the expected slopes of regression of the eventual follow-up values on the original block estimates;

• Slopes of less than unity indicate the presence of conditional biases, with blocks in the upper grade categories overvalued and the reverse applying to blocks evaluated as low grade.

• Block evaluations subject to conditional biases have lower efficiencies.
Professor Krige in 1996 proposed to define and measure the efficiency as follows:

\[
\text{Efficiency} = \frac{(BV-KV)}{BV} \text{ expressed as a percentage.}
\]

Where: \( BV = \) Block Variance (i.e. the variance of actual block values, calculated from a variogram), and

\( KV = \) Kriging Variance (i.e. the error variance of the respective estimates).
Efficiency of Block Evaluations

• An extensive study of some 70 cases by Professor Krige covering a wide range of spatial and data patterns used, indicated a correlation between kriging efficiency and the regression slope (actuals on estimates) of 87.5% (Krige 1996);

• Thus the slope (or the extent of conditional biases present) effectively incorporates all the major factors affecting the efficiency of block evaluations.

As highlighted by Professor Krige “The efficiency [of a block evaluation] can even be negative if KV > BV. Such a situation is ridiculous and the block evaluations will be worthless; yet the author [Professor Krige] encountered several such cases in practice, where the data accessed per block was inadequate”.

TEST FOR CONDITIONAL BIASES – REGRESSION SLOPE

BV = Block Variance, ie. variance of actual kriged blocks, computed from the variogram

KV = Respective Kriging Variance of kriged block estimates.

LM = Respective Lagrange multiplier for Ordinary Kriging.

Regression Slope \( = \frac{(BV - KV + |LM|)}{(BV - KV + 2|LM|)} \) ......................... (1)

Where only a global estimate of all blocks is practical, all blocks will be valued at the global or sub-domain mean i.e., \( KV = BV \) and Efficiency = 0

Substituting \( KV = BV \) into equation (1)

Regression Slope \( = \frac{|LM|}{2|LM|} = 0.5 \)

Thus a Regression slope less than 0.5 will always lead to a negative block efficiency (ie worthless kriged estimates).

This critical regression slope limit (0.5) should only be used to identify blocks which will result with negative efficiencies. Ideal Slopes are still > 0.95 (Krige, 1996).
The absence of conditional biases is unavoidably accompanied by some *smoothing* and it is a *fallacy* to use the data search routine for block evaluation in an endeavour to reduce or eliminate smoothing.
Theoretical Explanation:

The definition of the slope of the line of regression of actuals (Y) on estimated block values (X), i.e.

Slope = r \left( \frac{\sigma_y}{\sigma_x} \right)

If the Slope is to be unity, (i.e. Slope = \sim 1) for unbiased block estimates and (r) is less than unity, because estimates are never perfect:

Then \quad \left( \frac{\sigma_y}{\sigma_x} \right) > 1

i.e. the standard deviation (or variance) of the actual or real block values must be larger than that of the estimated block values

The gap between these two variances, i.e the Smoothing Effect can, therefore, only be reduced by increasing the correlation (r) between block estimates and actual values, i.e., by improving the efficiency of the estimation technique or by providing more data. No mathematical manoeuvering can achieve this objective.
Smoothing Effect of Kriging

• Various post-processing techniques are available to remove smoothing effects (e.g., Krige and Assibey-Bonsu, 1999; Journel et al, 2000) and should be applied only to block or panel estimates that are conditionally unbiased.
Part 3 - Conclusion

Professor Krige’s outstanding influence on the worldwide mining industry is visible everyday, as shown by the decision making processes followed by international mining companies.

Professor Krige’s Basic Points of Advice

1. Ensure data integrity;
2. Establish all the necessary geological and geostatistical models and parameters;
3. Select appropriate techniques and ensure optimal application: choose appropriate techniques and determine the search routine required to eliminate conditional biases. Use effective tools, including slope of regression and block efficiencies in this regard;
4. As blocks are mined out, do follow-up reconciliation studies to validate Mineral Resource estimates - the fundamental asset of mining companies;
5. Research new techniques and applications considering the above, but validate new techniques and confirm the practical advantages to be gained when they are applied in practice.
Conclusion (cotd.)

The industry seems to be going backwards in certain areas, with a widely spread misunderstanding of the causes and consequences of conditional biases. The following are some of the possible causes:

• In certain universities, as well as training provided elsewhere in the industry, geostatistics is taught using commercially available computer programs, the emphasis being how to use the programs;

• Unfortunately, this is what many mining companies expect: graduates or practitioners who are good at operating programs (“black box approach”). This does not give much time to teaching the fundamentals of geostatistics and the consequences of misusing the technology;

• What complicates matters is that, the universities rarely have large databases to demonstrate the strengths and weaknesses of methods in different environments, and research is by its own nature geared towards only development of theoretical geostatistics, often based on strong stationarity assumptions.
“… after half a century of phenomenal developments in geostatistics, conditional biases which gave birth to this subject, are still encountered in practical applications… the main concern is that this record will be tarnished by the all too ready acceptance [in certain cases] of estimates, which are still conditionally biased. For the future, I would like to see geostatistics continue to grow from strength to strength with new models, techniques and applications, but where these are all validated properly by way of [follow-up] checks to confirm the absence of biases and the practical advantages to be gained when they are applied in practice ” …Professor Danie Krige

“Methods are many,
Principles are few.
Methods always change,
Principles never do.” …Warren Wiersbe
A Final Personal Tribute

• I am privileged to have had an association with Professor Krige for more than two decades, first as his student during my doctorate programme at the University of the Witwatersrand, and secondly, as my mentor and ‘father figure’ over these years. He has been an immense source of inspiration, shaping my career in the same way as he did with so many others that he educated;

• His focus was always on what he could give to people rather than what he could get out of them;

• Irrespective of his international accomplishments, he was a very humble man of high integrity and treated everyone with respect;

• Professor Krige has demonstrated that true success in life, has nothing to do with what you gain in life or accomplish for yourself. It is what you do for others that matters.

• It has been said that we make a living by what we get, but we make a life by what we give. May his legacy live on forever!
You Made the World a Better Place

Professor Daniel Gerhardus Krige
26 August 1919 – 2 March 2013